6-3 Dividing Polynomials

Main Ideas

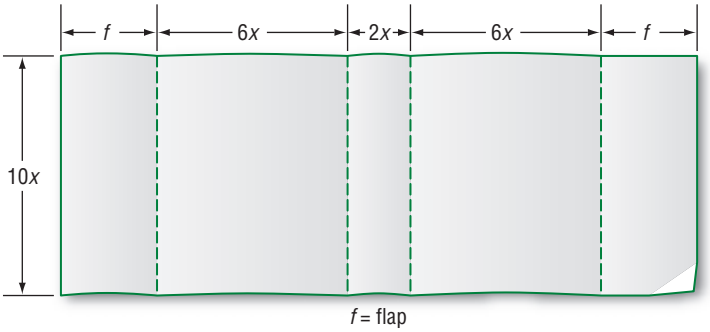
- Divide polynomials using long division.
- Divide polynomials using synthetic division.

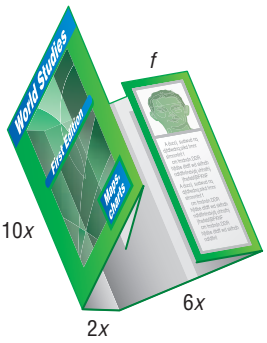
New Vocabulary

synthetic division

GET READY for the Lesson

Arianna needed $140x^2 + 60x$ square inches of paper to make a book jacket $10x$ inches tall. In figuring the area she needed, she allowed for a front and back flap. If the spine of the book jacket is $2x$ inches, and the front and back of the book jacket are $6x$ inches, how wide are the front and back flaps? You can use a quotient of polynomials to help you find the answer.





Use Long Division In Lesson 6-1, you learned to divide monomials. You can divide a polynomial by a monomial by using those same skills.

EXAMPLE 1 Divide a Polynomial by a Monomial

Simplify $\dfrac{4x^3y^2 + 8xy^2 - 12x^2y^3}{4xy}$.

$$\frac{4x^3y^2 + 8xy^2 - 12x^2y^3}{4xy} = \frac{4x^3y^2}{4xy} + \frac{8xy^2}{4xy} - \frac{12x^2y^3}{4xy}$$ Sum of quotients

$$= \frac{4}{4} \cdot x^{3-1}y^{2-1} + \frac{8}{4} \cdot x^{1-1}y^{2-1} - \frac{12}{4} \cdot x^{2-1}y^{3-1}$$ Divide.

$$= x^2y + 2y - 3xy^2$$ $x^{1-1} = x^0$ or 1

CHECK Your Progress

Simplify.

1A. $\dfrac{9x^2y^3 - 15xy^2 + 12xy^3}{3xy^2}$

1B. $\dfrac{16a^5b^3 + 12a^3b^4 - 20ab^5}{4ab^3}$

1C. $(20c^4d^2f - 16cf + 4cdf)(4cdf)^{-1}$

1D. $(18x^2y + 27x^3y^2z)(3xy)^{-2}$

You can use a process similar to long division to divide a polynomial by a polynomial with more than one term. The process is known as the *division algorithm*. When doing the division, remember that you can only add or subtract like terms.

EXAMPLE Division Algorithm

2 **Use long division to find $(z^2 + 2z - 24) \div (z - 4)$.**

$$\begin{array}{r} z \\ z-4\overline{)\,z^2+2z-24} \\ \underline{(-)z^2-4z} \\ 6z-24 \\ \underline{(-)6z-24} \\ 0 \end{array}$$

$z(z - 4) = z^2 - 4z$

$2z - (-4z) = 6z$

$$\begin{array}{r} z+6 \\ z-4\overline{)\,z^2+2z-24} \\ \underline{(-)z^2-4z} \\ 6z-24 \\ \underline{(-)6z-24} \\ 0 \end{array}$$

The quotient is $z + 6$. The remainder is 0.

CHECK Your Progress

Use long division to find each quotient.

2A. $(x^2 + 7x - 30) \div (x - 3)$ **2B.** $(x^2 - 13x + 12) \div (x - 1)$

Just as with the division of whole numbers, the division of two polynomials may result in a quotient with a remainder. Remember that $9 \div 4 = 2 + \text{R1}$ and is often written as $2\frac{1}{4}$. The result of a division of polynomials with a remainder can be written in a similar manner.

STANDARDIZED TEST EXAMPLE Quotient with Remainder

3 Which expression is equal to $(t^2 + 3t - 9)(5 - t)^{-1}$?

A $t + 8 - \frac{31}{5 - t}$

B $-t - 8$

C $-t - 8 + \frac{31}{5 - t}$

D $-t - 8 - \frac{31}{5 - t}$

Read the Test Item

Since the second factor has an exponent of -1, this is a division problem.

$(t^2 + 3t - 9)(5 - t)^{-1} = \frac{t^2 + 3t - 9}{5 - t}$

Test-Taking Tip

You may be able to eliminate some of the answer choices by substituting the same value for t in the original expression and the answer choices and evaluating.

Solve the Test Item

$$\begin{array}{r} -t-8 \\ -t+5\overline{)\,t^2+3t-9} \\ \underline{(-)t^2-5t} \\ 8t-9 \\ \underline{(-)8t-40} \\ 31 \end{array}$$

For ease in dividing, rewrite $5 - t$ as $-t + 5$.

$-t(-t + 5) = t^2 - 5t$

$3t - (-5t) = 8t$

$-8(-t + 5) = 8t - 40$

Subtract. $-9 - (-40) = 31$

The quotient is $-t - 8$, and the remainder is 31. Therefore, $(t^2 + 3t - 9)(5 - t)^{-1} = -t - 8 + \frac{31}{5 - t}$. The answer is C.

CHECK Your Progress

3. Which expression is equal to $(r^2 + 5r + 7)(1 - r)^{-1}$?

F $-r - 6 + \frac{13}{1 - r}$ **G** $r + 6$ **H** $r - 6 + \frac{13}{1 - r}$ **J** $r + 6 - \frac{13}{1 - r}$

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Use Synthetic Division

Synthetic division is a simpler process for dividing a polynomial by a binomial. Suppose you want to divide $5x^3 - 13x^2 + 10x - 8$ by $x - 2$ using long division. Compare the coefficients in this division with those in Example 4.

$$
\begin{array}{r}
5x^2 - 3x + 4 \\
x - 2 \overline{)\,5x^3 - 13x^2 + 10x - 8} \\
\underline{(-)5x^3 - 10x^2} \\
-3x^2 + 10x \\
\underline{(-)-3x^2 + 6x} \\
4x - 8 \\
\underline{(-)4x - 8} \\
0
\end{array}
$$

EXAMPLE Synthetic Division

4 **Use synthetic division to find $(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$.**

Step 1 Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients as shown at the right.

$$
\begin{array}{cccc}
5x^3 & -13x^2 & +10x & -8 \\
\downarrow & \downarrow & \downarrow & \downarrow \\
5 & -13 & 10 & -8
\end{array}
$$

Step 2 Write the constant r of the divisor $x - r$ to the left. In this case, $r = 2$. Bring the first coefficient, 5, down.

$$
\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -8 \\
\hline
 & 5 & & &
\end{array}
$$

Step 3 Multiply the first coefficient by r: $2 \cdot 5 = 10$. Write the product under the second coefficient. Then add the product and the second coefficient: $-13 + 10 = -3$.

$$
\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -8 \\
 & & 10 & & \\
\hline
 & 5 & -3 & &
\end{array}
$$

Step 4 Multiply the sum, -3, by r: $2(-3) = -6$. Write the product under the next coefficient and add: $10 + (-6) = 4$.

$$
\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -8 \\
 & & & 10 & -6 \\
\hline
 & 5 & -3 & 4 &
\end{array}
$$

Step 5 Multiply the sum, 4, by r: $2 \cdot 4 = 8$. Write the product under the next coefficient and add: $-8 + 8 = 0$. The remainder is 0.

$$
\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -8 \\
 & & & 10 & -6 & 8 \\
\hline
 & 5 & -3 & 4 &
\end{array}
$$

The numbers along the bottom row are the coefficients of the quotient. Start with the power of x that is one less than the degree of the dividend. Thus, the quotient is $5x^2 - 3x + 4$.

CHECK Your Progress **Use synthetic division to find each quotient.**

4A. $(2x^3 + 3x^2 - 4x + 15) \div (x + 3)$ **4B.** $(3x^3 - 8x^2 + 11x - 14) \div (x - 2)$

To use synthetic division, the divisor must be of the form $x - r$. If the coefficient of x in a divisor is not 1, you can rewrite the division expression so that you can use synthetic division.

EXAMPLE Divisor with First Coefficient Other than 1

5 **Use synthetic division to find $(8x^4 - 4x^2 + x + 4) \div (2x + 1)$.**

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{8x^4 - 4x^2 + x + 4}{2x + 1} = \frac{(8x^4 - 4x^2 + x + 4) \div 2}{(2x + 1) \div 2}$$ Divide numerator and denominator by 2.

$$= \frac{4x^4 - 2x^2 + \frac{1}{2}x + 2}{x + \frac{1}{2}}$$ Simplify the numerator and denominator.

(continued on the next page)

Since the numerator does not have an x^3-term, use a coefficient of 0 for x^3.

$x - r = x + \frac{1}{2}$, so $r = -\frac{1}{2}$.

$$\begin{array}{r|cccc|c} -\frac{1}{2} & 4 & 0 & -2 & \frac{1}{2} & 2 \\ & & -2 & 1 & \frac{1}{2} & -\frac{1}{2} \\ \hline & 4 & -2 & -1 & 1 & \frac{3}{2} \end{array}$$

The result is $4x^3 - 2x^2 - x + 1 + \dfrac{\frac{3}{2}}{x + \frac{1}{2}}$. Now simplify the fraction.

$$\frac{\frac{3}{2}}{x + \frac{1}{2}} = \frac{3}{2} \div \left(x + \frac{1}{2}\right) \quad \text{Rewrite as a division expression.}$$

$$= \frac{3}{2} \div \frac{2x + 1}{2} \quad x + \tfrac{1}{2} = \tfrac{2x}{2} + \tfrac{1}{2} = \tfrac{2x+1}{2}$$

$$= \frac{3}{2} \cdot \frac{2}{2x + 1} \quad \text{Multiply by the reciprocal.}$$

$$= \frac{3}{2x + 1} \quad \text{The solution is } 4x^3 - 2x^2 - x + 1 + \frac{3}{2x + 1}.$$

CHECK Divide using long division.

$$\begin{array}{r} 4x^3 - 2x^2 - x + 1 \\ 2x + 1 \overline{)\,8x^4 + 0x^3 - 4x^2 + x + 4} \\ \underline{(-)8x^4 + 4x^3} \\ -4x^3 - 4x^2 \\ \underline{(-)-4x^3 - 2x^2} \\ -2x^2 + x \\ \underline{(-)-2x^2 - x} \\ 2x + 4 \\ \underline{(-)2x + 1} \\ 3 \end{array}$$

The result is $4x^3 - 2x^2 - x + 1 + \dfrac{3}{2x + 1}$. ✓

CHECK Your Progress **Use synthetic division to find each quotient.**

5A. $(3x^4 - 5x^3 + x^2 + 7x) \div (3x + 1)$ **5B.** $(8y^5 - 2y^4 - 16y^2 + 4) \div (4y - 1)$

CHECK Your Understanding

Example 1 (p. 325)

Simplify.

1. $\dfrac{6xy^2 - 3xy + 2x^2y}{xy}$

2. $(5ab^2 - 4ab + 7a^2b)(ab)^{-1}$

3. BAKING The number of cookies produced in a factory each day can be estimated by $C(w) = -w^2 + 16w + 1000$, where w is the number of workers and C is the number of cookies produced. Divide to find the average number of cookies produced per worker.

Examples 2, 4 (pp. 326–327)

Simplify.

4. $(x^2 - 10x - 24) \div (x + 2)$

5. $(3a^4 - 6a^3 - 2a^2 + a - 6) \div (a + 1)$

6. $(z^5 - 3z^2 - 20) \div (z - 2)$

7. $(x^3 + y^3) \div (x + y)$

8. $\dfrac{x^3 + 13x^2 - 12x - 8}{x + 2}$

9. $(b^4 - 2b^3 + b^2 - 3b + 4)(b - 2)^{-1}$

Example 3 (p. 326)

10. STANDARDIZED TEST PRACTICE Which expression is equal to $(x^2 - 4x + 6)(x - 3)^{-1}$?

A $x - 1$ **B** $x - 1 + \frac{3}{x-3}$ **C** $x - 1 - \frac{3}{x-3}$ **D** $-x + 1 - \frac{3}{x-3}$

Example 5 (pp. 327–328)

Simplify.

11. $(12y^2 + 36y + 15) \div (6y + 3)$ **12.** $\frac{9b^2 + 9b - 10}{3b - 2}$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–16	1
17–22	2, 4
23–28	3, 4
29–34	2, 3, 5

Simplify.

13. $\frac{9a^3b^2 - 18a^2b^3}{3a^2b}$ **14.** $\frac{5xy^2 - 6y^3 + 3x^2y^3}{xy}$

15. $(28c^3d - 42cd^2 + 56cd^3) \div (14cd)$ **16.** $(a^3b^2 - a^2b + 2a)(-ab)^{-1}$

17. $(x^3 - 4x^2) \div (x - 4)$ **18.** $(x^3 - 27) \div (x - 3)$

19. $(b^3 + 8b^2 - 20b) \div (b - 2)$ **20.** $(g^2 + 8g + 15)(g + 3)^{-1}$

21. $\frac{y^3 + 3y^2 - 5y - 4}{y + 4}$ **22.** $\frac{m^3 + 3m^2 - 7m - 21}{m + 3}$

23. $(t^5 - 3t^2 - 20)(t - 2)^{-1}$ **24.** $(y^5 + 32)(y + 2)^{-1}$

25. $(2c^3 - 3c^2 + 3c - 4) \div (c - 2)$ **26.** $(2b^3 + b^2 - 2b + 3)(b + 1)^{-1}$

27. $\frac{x^5 - 7x^3 + x + 1}{x + 3}$ **28.** $\frac{3c^5 + 5c^4 + c + 5}{c + 2}$

29. $\frac{4x^3 + 5x^2 - 3x + 1}{4x + 1}$ **30.** $\frac{x^3 - 3x^2 + x - 3}{x^2 + 1}$

31. $(6t^3 + 5t^2 + 9) \div (2t + 3)$ **32.** $\frac{x^4 + x^2 - 3x + 5}{x^2 + 2}$

33. $\frac{2x^4 + 3x^3 - 2x^2 - 3x - 6}{2x + 3}$ **34.** $\frac{6x^4 + 5x^3 + x^2 - 3x + 1}{3x + 1}$

35. ENTERTAINMENT A magician gives these instructions to a volunteer.

- Choose a number and multiply it by 4.
- Then add the sum of your number and 15 to the product you found.
- Now divide by the sum of your number and 3.

What number will the volunteer always have at the end? Explain.

Larry Dale Gordon/Getty Images

BUSINESS **For Exercises 36 and 37, use the following information.**

The number of sports magazines sold can be estimated by $n = \frac{3500a^2}{a^2 + 100}$, where a is the amount of money spent on advertising in hundreds of dollars and n is the number of subscriptions sold.

36. Perform the division indicated by $\frac{3500a^2}{a^2 + 100}$.

37. About how many subscriptions will be sold if \$1500 is spent on advertising?

Real-World Career

Cost Analyst

Cost analysts study and write reports about the factors involved in the cost of production.

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PHYSICS **For Exercises 38–40, suppose an object moves in a straight line so that, after t seconds, it is $t^3 + t^2 + 6t$ feet from its starting point.**

38. Find the distance the object travels between the times $t = 2$ and $t = x$, where $x > 2$.

39. How much time elapses between $t = 2$ and $t = x$?

40. Find a simplified expression for the average speed of the object between times $t = 2$ and $t = x$.

H.O.T. Problems

41. OPEN ENDED Write a quotient of two polynomials such that the remainder is 5.

42. REASONING Review any of the division problems in this lesson. What is the relationship between the degrees of the dividend, the divisor, and the quotient?

43. FIND THE ERROR Shelly and Jorge are dividing $x^3 - 2x^2 + x - 3$ by $x - 4$. Who is correct? Explain your reasoning.

Shelly

4	1	−2	1	−3
		4	−24	100
	1	−6	25	−103

Jorge

4	1	−2	1	−3
		4	8	36
	1	2	9	33

44. CHALLENGE Suppose the result of dividing one polynomial by another is $r^2 - 6r + 9 - \frac{1}{r-3}$. What two polynomials might have been divided?

45. *Writing in Math* Use the information on page 325 to explain how you can use division of polynomials in manufacturing. Include the dimensions of the piece of paper that the publisher needs, the formula from geometry that applies to this situation, and an explanation of how to use division of polynomials to find the width of the flap.

EXTRA PRACTICE
See pages 903, 931.
MathOnline
Self-Check Quiz at algebra2.com

STANDARDIZED TEST PRACTICE

46. ACT/SAT What is the remainder when $x^3 - 7x + 5$ is divided by $x + 3$?

A −11 C 1

B −1 D 11

47. REVIEW If $i = \sqrt{-1}$, then $5i(7i) =$

F 70 H −35

G 35 J −70

Spiral Review

Simplify. (Lesson 6-2)

48. $(2x^2 - 3x + 5) - (3x^2 + x - 9)$

49. $y^2z(y^2z^3 - yz^2 + 3)$

50. $(y + 5)(y - 3)$

51. $(a - b)^2$

52. ASTRONOMY Earth is an average of 1.5×10^{11} meters from the Sun. Light travels at 3×10^8 meters per second. About how long does it take sunlight to reach Earth? (Lesson 6-1)

GET READY for the Next Lesson

PREREQUISITE SKILL **Given $f(x) = x^2 - 5x + 6$, find each value.** (Lesson 2-1)

53. $f(-2)$ **54.** $f(2)$ **55.** $f(2a)$ **56.** $f(a + 1)$